PHASE COEXISTENCE FOR LATTICE MODELS
– MARK KAC SEMINAR –

The theory of equilibrium crystals dates back to the work of Wulff in the early 20th century. Following a thermodynamic approach, Wulff predicted the equilibrium crystal shapes as the minimizers of the interfacial energy. By contrast, the microscopic theory aims at deriving the interfacial free energy and the corresponding variational problem from interatomic forces. This series of lectures intends to survey recent developments in the study of phase coexistence starting at the microscopic level from lattice models with finite range interactions.

First Lecture. *Wulff Construction*

This introductory lecture will be devoted to the mathematical derivation of the thermodynamic theory of phase segregation and the justification of Wulff construction in three dimensions. After looking at the history of the problem, we will focus on the Ising model and present in this framework a general scheme, dubbed the L^1-*approach*, which has been devised to deal with three dimensional models. The basic concepts of surface tension and coarse-graining will be introduced.

In the second part of the lecture, extension of these results to other lattice models will be discussed including the Kac model and a class of models in the Pirogov-Sinai theory. Finally, the influence of boundary interaction will be taken into account leading to the Winterbottom construction.

Second Lecture. *Beyond the thermodynamic predictions*

Renormalization plays an important role in the derivation of Wulff construction. In the first part of this lecture, coarse-grained estimates will be detailed in the framework of the Ising model. As a consequence, the L^1-approach can be extended up to the critical temperature.

The second part of the talk will be devoted to interface structure. The L^1-approach was initiated in order to bypass the complexity of the microscopic configurations, but, unlike the two-dimensional case, no precise controls on the interface can be inferred from the L^1-estimates. We are going to present enhancements of the L^1-approach which allow to prove stability of the interface with respect to the Hausdorff distance. The issue of interface fluctuation will also be discussed, in particular the rigidity of the flat facets in the low temperature Wulff shapes.

Third Lecture. *On the slow relaxation of Glauber dynamics*

In general, the relaxation of non-equilibrium macroscopic crystals involves time scales that are unrealistically long for experiments and the crystal shapes commonly observed never reach equilibrium. Thus the study of the kinetics of growth is a relevant issue to further understand the phase coexistence phenomena. We will present some results on the slow convergence to equilibrium of the Glauber dynamics in the phase transition regime. As there is a strong interplay between equilibrium and dynamical properties of particles systems, the slowdown of the dynamics will be related to the results on equilibrium phase coexistence presented in the previous lectures.