Title: *Gibbs measures on combinatorial structures.*

Speaker: *Shankar Bhamidi, UNC Chapel Hill*

Abstract:

We shall report on some work in progress in understanding various aspects of Gibbs distributions on combinatorial structures and shall describe why such problems are of crucial importance, both in practice and for the rich mathematical theory that one can develop. We first look at the space of all unordered trees, labeled trees on N nodes, say $\mathcal{T}_N$. For each tree $\mathbf{t} \in \mathcal{T}_n$ consider the case where the Hamiltonian consists of the number of leaves namely

$$H(\mathbf{t}) = \beta \# \text{ of leaves in } \mathbf{t}$$

for some constant β. Now consider the probability measure on $\mathcal{T}_n$ given by

$$p_N(\mathbf{t}) = \frac{\exp(\beta H(\mathbf{t}))}{Z_N(\beta)}$$

Note that as $\beta \to \infty$ we shall end up with star like trees while as $\beta \to -\infty$ we shall end up with path like trees. We shall identify these regimes of this model (star like, continuum random tree like and path like regimes) and exhibit where the phase transitions occur. We shall also show how these methods can be used to get large deviation results for the uniform random tree model as well as how general theory implies the convergence of the spectral distribution of the adjacency matrix of a tree $\mathbf{t} \sim p_N(\cdot)$ to a non-degenerate non-random distribution whose atoms are dense on the real line.

We shall then report on some recent work on a related model, the exponential random graph model. Let us first put this problem in the context of recent research and describe why such problems are crucial. A variety of random graph models have been developed in recent years to study a range of problems on networks, driven by the wide availability of data from many social, telecommunication, biochemical and other networks. The exponential random graph model is a key model, extensively used in the sociology literature. This model seeks to incorporate in random graphs the notion of reciprocity, that is, the larger than expected number of triangles and other small subgraphs. Sampling from these distributions is crucial for parameter estimation hypothesis testing, and more generally for understanding basic features of the network model itself. In practice sampling is typically carried out using Markov chain Monte Carlo, in particular either the Glauber dynamics or the Metropolis-Hasting procedure.

In the talk we characterize the high and low temperature regimes of the exponential random graph model. We establish that in the high temperature regime the mixing time of the Glauber dynamics is $\Theta(n^2 \log n)$, where n is the number of vertices in the graph; in contrast, we show that in the low temperature regime the mixing is exponentially slow for any local Markov chain. Our results, moreover, give a rigorous basis for criticisms made of such models. In the high temperature regime, where sampling with MCMC is possible, we show that any finite collection of edges are asymptotically independent; thus, the model does not possess the desired reciprocity property, and is not appreciably different from the Erdős-Rényi random graph.